Space charge distortions

May 11 2016

Carlos Perez

Outlook:

- How to compute distortions
- R-Distortions ALICE/PHENIX
- Charge density from FP

Motivation

- The ions drifting slowly in the TPC can lead to a significant accumulation of charge that ultimately distort the E and B fields.
- The resulting field distortions modify the electron drift lines, introducing drift distortions that have to be corrected.
- Depending upon fluctuations, the residuals might impact significantly the tracking resolution.
- Quantification of the effect on tracking resolution is the objective of this study.

Methodology

Space Charge Distribution

↓

E (and B) Field Distortions

↓

Drift distortions

Space charge

Two sources:

- [1] prompt contribution of the gas ionisation by charge particles crossing the TPC
- [2] delayed contribution due to ion back flow from the GEM readout system

Space charge distribution: Method 1

Toy model: taken from ALICE TDR

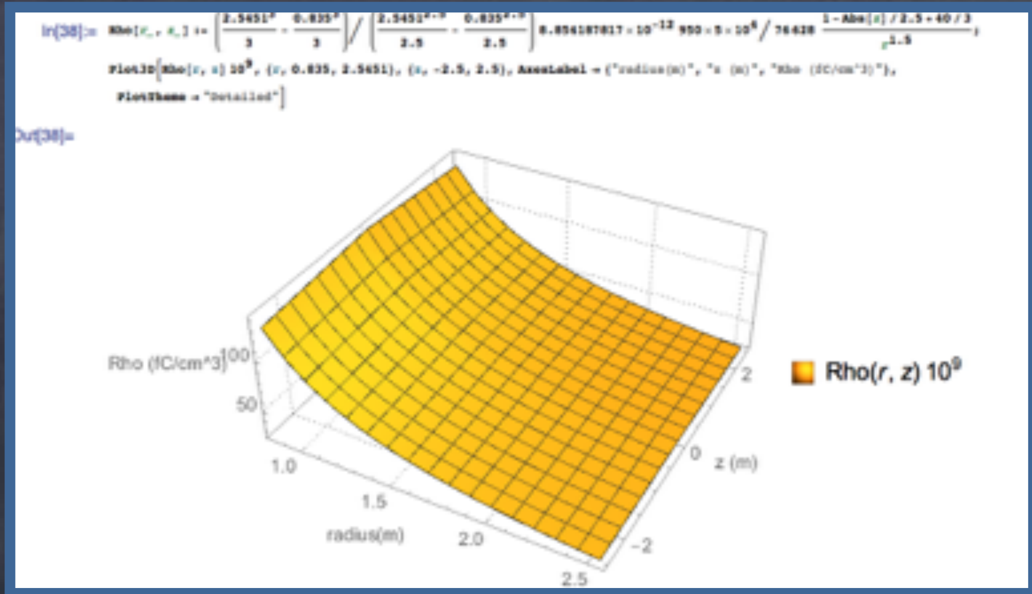
$$\rho(r\_, z\_) := A\left(\frac{1 - b\,z + c\,\epsilon}{f_d\,r^d}\right)$$

1. Proportionality to the primary ionisation (i.e. local track density in a collision) r^-2 dependence and Z drift velocity
2. Back flow dependence as CTE in Z direction

Space charge density in the TPC volume

$$\rho(r\_, z\_) := A\left(\frac{1 - b\,z + c\,\epsilon}{f_d\,r^d}\right)$$

- A = [G] x [M] x [R] x [e_0] / 76628 [in C/m]
  - e_0 (=8.85e-12): vacuum permittivity [in As/(Vm)]
  - G (=1): gas factor (prim ioniz. / drift velocity)
  - M (=950): nominal event multiplicity
  - R (=5e4): total interaction rate [in Hz]
- b (=1/2.5): 1/DriftLength [in 1/m]
- c*e (=2/3*20)
- d (=2 for STAR f_d=1; =1.5 for ALCE)



Using ALICE parameters into Toy Function

Alice TPC upgrade TDR

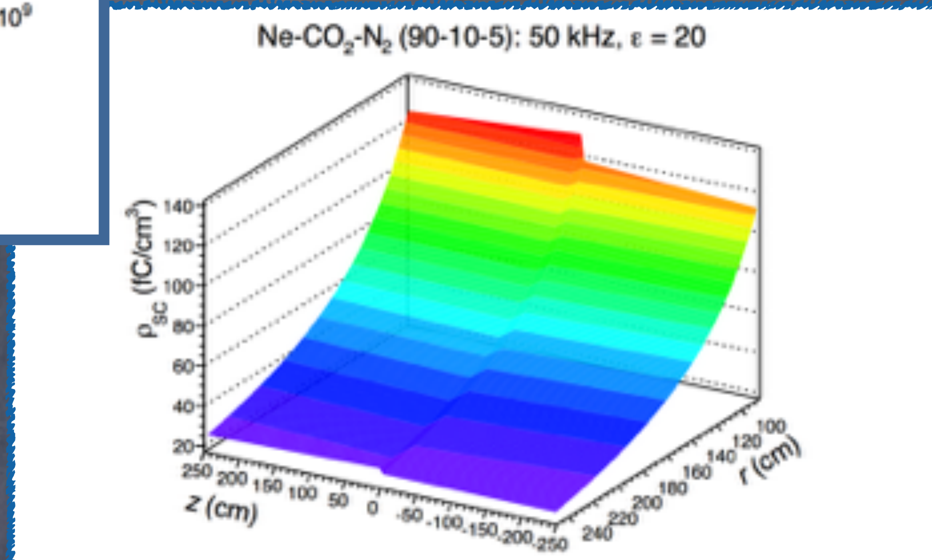


Figure 7.7: Average space charge density for Ne-CO$_2$-N$_2$ (90-10-5), R_{int} = 50 kHz and ε = 20.

M: 950

IntR: 50kHz

E: 20 (i.e. 1% in a residual gain of 2000)

Space charge distribution: Method 2

Toy simulation:

1. Detailed description of ionisation in gas and transport of each ion/electron + ion black flow.
2. More details on this method at the end of the presentation

Simulated here, but ultimately computed from data

Space Charge Distribution

↓

E (and B) Field Distortions

↓

Drift distortions

Simulated here, but ultimately computed from data

Space Charge Distribution

- Laplace formalism for superposition of charges (Tom's slides or backup)

E (and B) Field Distortions

Drift distortions

Simulated here, but ultimately computed from data

Space Charge Distribution

- Laplace formalism for superposition of charges (Tom's slides or backup)

E (and B) Field Distortions

- Langevin formalism up to 2nd order

Drift distortions

Langevin Eq:

charge of the drifting particle

Friction (K>0)

$$m\frac{d\vec{u}}{dt} = qe\vec{E} + qe\left[\vec{u}\times\vec{B}\right] - K\vec{u}$$

drift velocity

EB force

Solution:

$t \gg m/K$ Adiabatic approx.

$\frac{d\vec{u}}{dt} = 0$ Steady state

$$\vec{u} = \frac{\mu\left|\vec{E}\right|}{1+\omega^2\tau^2}\left[\hat{E} + \omega\tau\left(\hat{E}\times\hat{B}\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}\right]$$

scalar mobility of the electric field

$\mu = \frac{qe}{K}$

mean interaction time between drifting electrons and atoms from the gas

cyclotron frequency for electron

$\omega\tau = q\mu B$

Drift velocity in cartesian coordinates

$$u_x = \frac{\mu\left|\vec{E}\right|}{1+\omega^2\tau^2}\left[\hat{E}_x + \omega\tau\left(\hat{E}_y\hat{B}_z - \hat{E}_z\hat{B}_y\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}_x\right]$$

$$u_y = \frac{\mu\left|\vec{E}\right|}{1+\omega^2\tau^2}\left[\hat{E}_y + \omega\tau\left(\hat{E}_z\hat{B}_x - \hat{E}_x\hat{B}_z\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}_y\right]$$

$$u_z = \frac{\mu\left|\vec{E}\right|}{1+\omega^2\tau^2}\left[\hat{E}_z + \omega\tau\left(\hat{E}_x\hat{B}_y - \hat{E}_y\hat{B}_x\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}_z\right]$$

We can compute the path integral of the drifting electron

$$\delta_x = \int u_x\, dt = \int \frac{u_x}{u_z}\frac{dz}{dt}\, dt = \int \frac{u_x}{u_z}\, dz$$

$$\delta_y = \int \frac{u_y}{u_z}\, dz \qquad \delta_z = \int \frac{u_z}{u_0}\, dz$$

TPC case: Ez >> Ex,Ey Bz >> Bx,By

$$u_x = \frac{\mu \left|\vec{E}\right|}{1+\omega^2\tau^2}\left[\hat{E}_x + \omega\tau\left(\hat{E}_y\hat{B}_z - \hat{E}_z\hat{B}_y\right) + \omega^2\tau^2\left(\hat{E}\cdot\hat{B}\right)\hat{B}_x\right]$$

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Second order expansion:

$$\hat{E}_x \approx \frac{\hat{E}_x}{E_z}$$

$$\hat{E}_z \approx 1 - \frac{1}{2}\hat{E}_x^2 - \frac{1}{2}\hat{E}_y^2$$

$$\frac{u_x}{u_z} = \frac{1}{1+\omega^2\tau^2}\frac{E_x}{E_z} + \frac{\omega\tau}{1+\omega^2\tau^2}\frac{E_y}{E_z} - \frac{\omega\tau}{1+\omega^2\tau^2}\frac{B_y}{B_z} + \frac{\omega^2\tau^2}{1+\omega^2\tau^2}\frac{B_x}{B_z}$$

$$\frac{u_y}{u_z} = \frac{1}{1+\omega^2\tau^2}\frac{E_y}{E_z} - \frac{\omega\tau}{1+\omega^2\tau^2}\frac{E_x}{E_z} + \frac{\omega\tau}{1+\omega^2\tau^2}\frac{B_x}{B_z} + \frac{\omega^2\tau^2}{1+\omega^2\tau^2}\frac{B_y}{B_z}$$

TPC case: Ez >> Ex,Ey Bz >> Bx,By

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$$\delta_x = c_0\int\frac{E_x}{E_z}\,dz + c_1\int\frac{E_y}{E_z}\,dz - c_1\int\frac{B_y}{B_z}\,dz + c2\int\frac{B_x}{B_z}\,dz$$

$$\delta_y = c_0\int\frac{E_y}{E_z}\,dz - c_1\int\frac{E_x}{E_z}\,dz + c_1\int\frac{B_x}{B_z}\,dz + c2\int\frac{B_y}{B_z}\,dz$$

First Calculations

ALICE reproduction

ALICE-TDR-016

CERN-LHCC-2013-020
March 3, 2014

Technical Design Report

for the

Upgrade of the
ALICE Time Projection Chamber

The ALICE Collaboration*

CERN-LHCC-2013-020 / ALICE-TDR-016
03/03/2014

*See list of authors in App. C

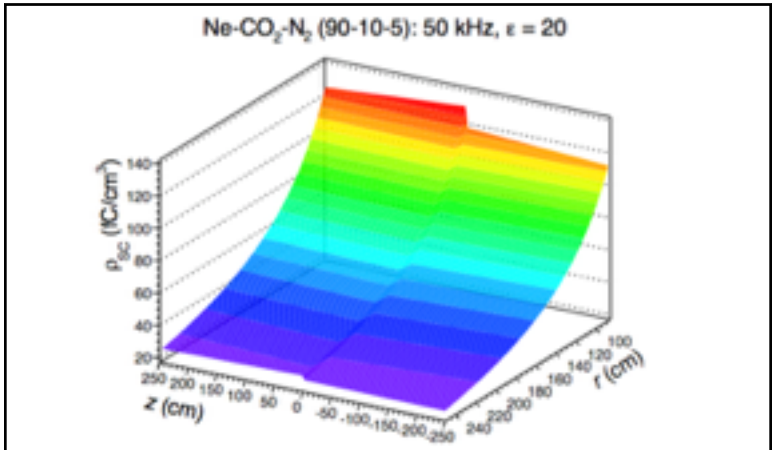
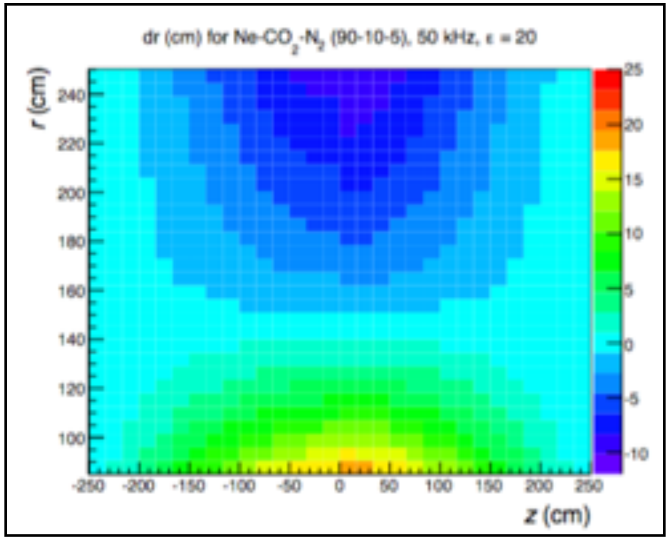


Figure 7.7: Average space charge density for Ne-CO$_2$-N$_2$ (90-10-5), R_{int} = 50 kHz and ε = 20.



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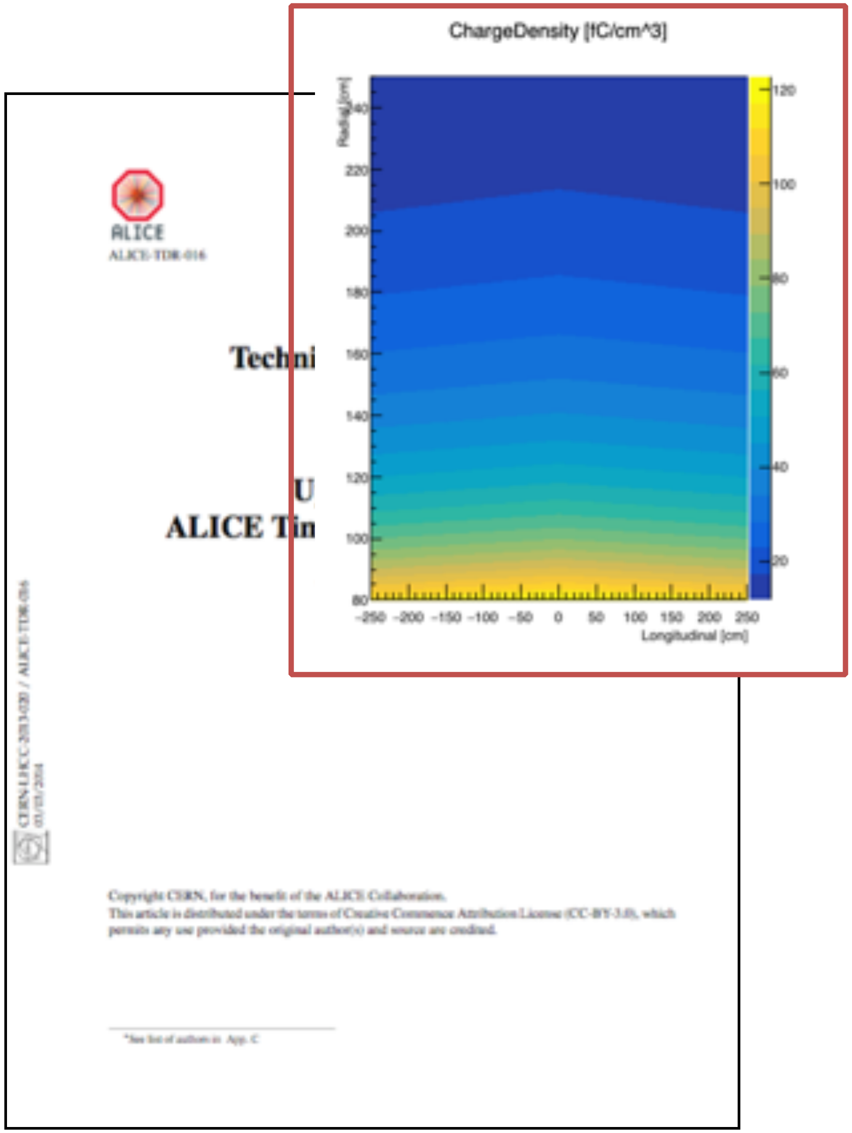
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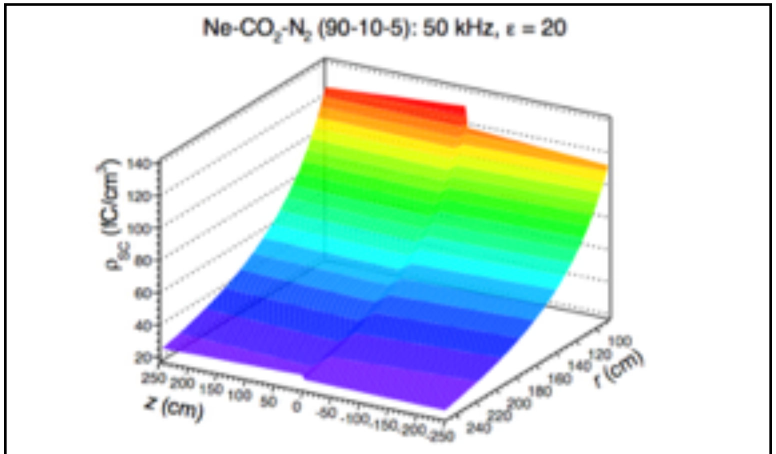

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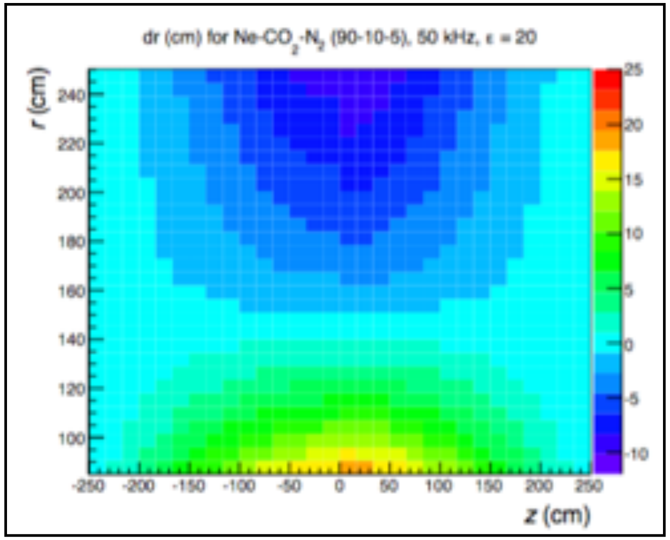

ALICE reproduction


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ALICE-TDR-016
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March 3, 2014

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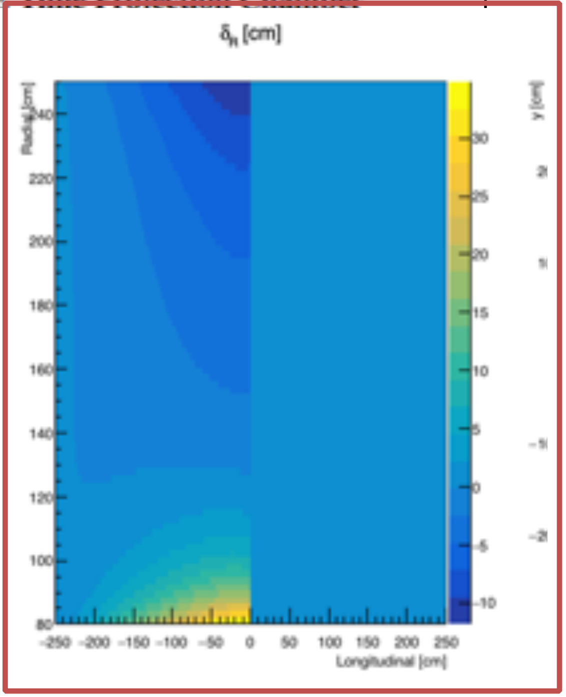

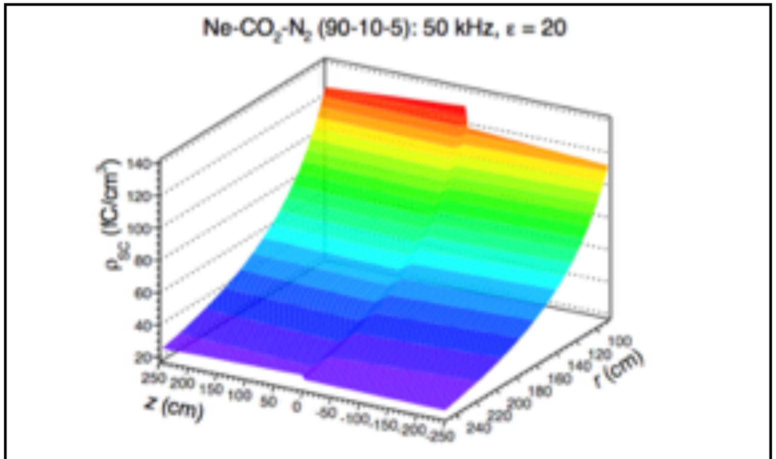

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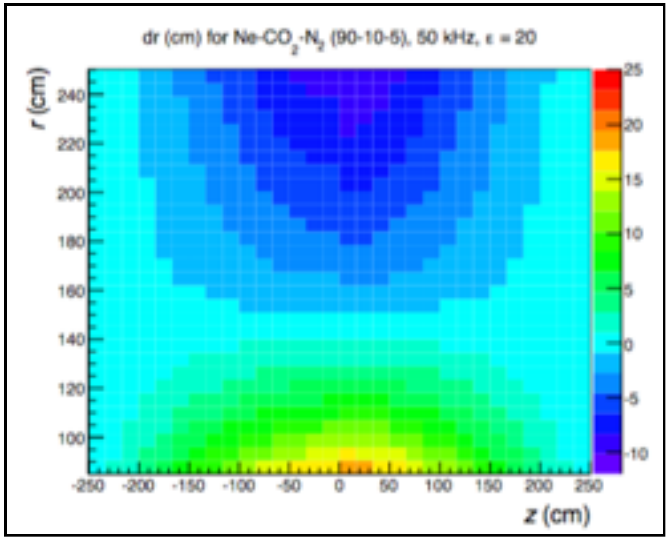
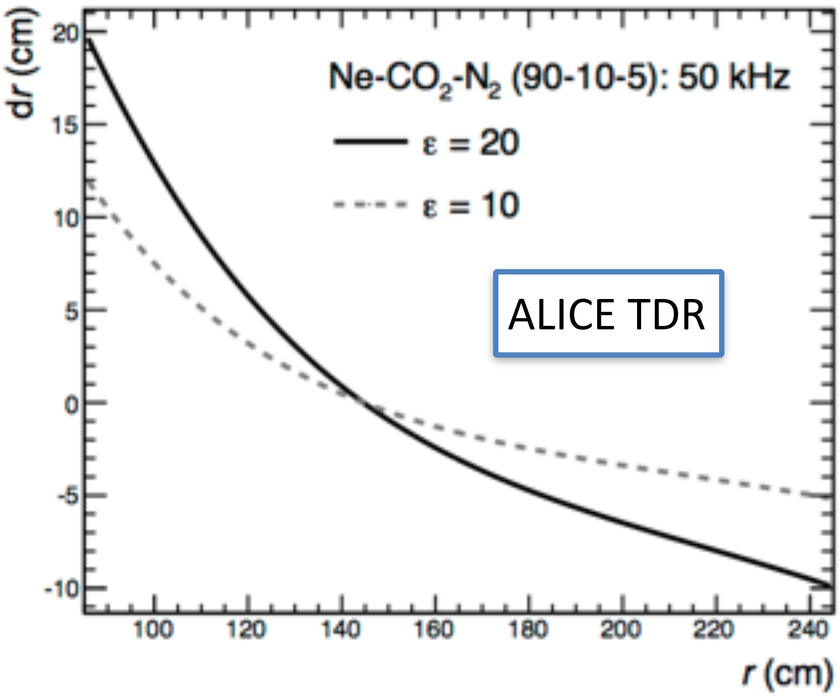
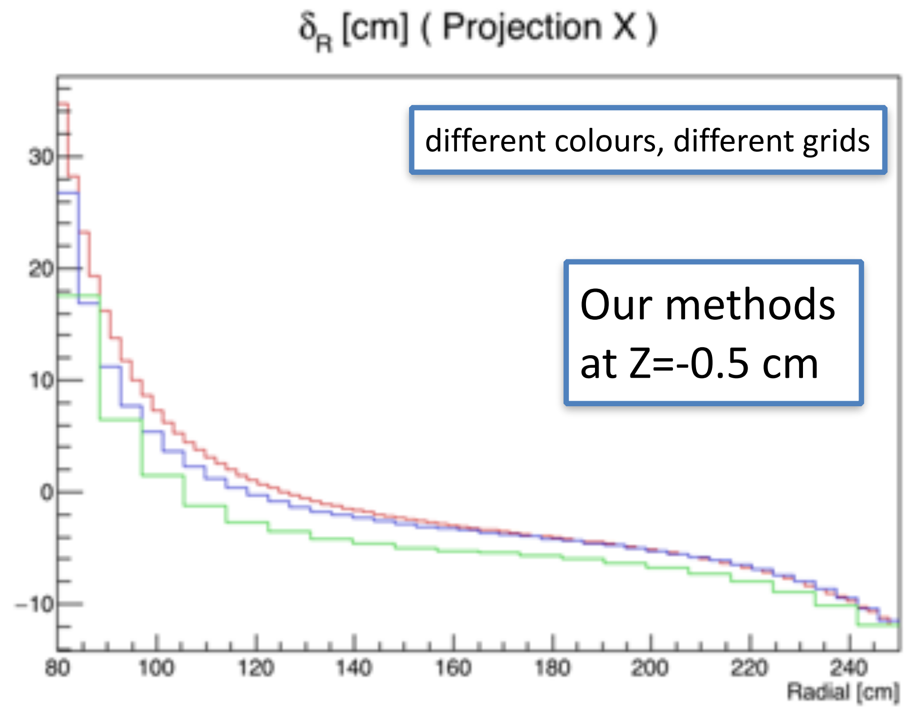


Dr detailed shape comparison





Quantitatively close, but not quite the right shape

Source of incongruence:

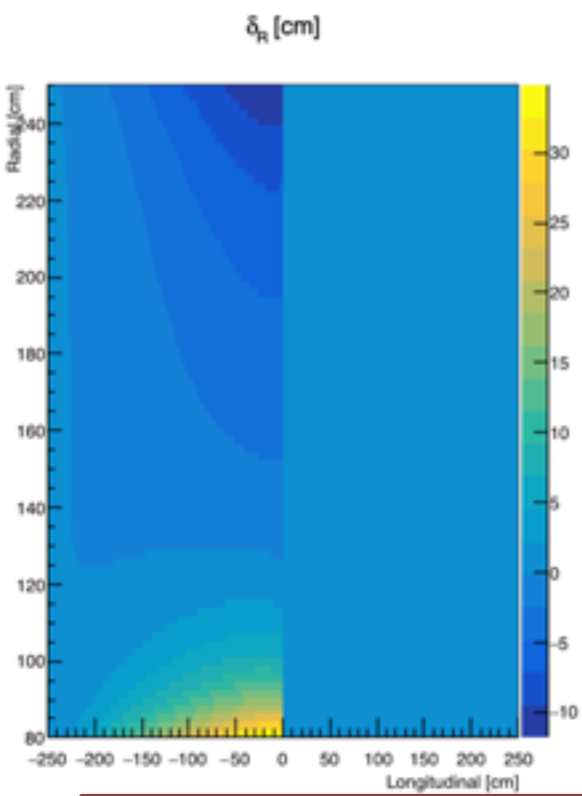
- We do Laplace expansion up to 15th order (ALICE 30th)
- We probe Dr at z=-0.5 cm (ALICE gets it at z=0)
- We use 1/r^2 in ICD (ALICE used 1/r^1.5 for TDR)

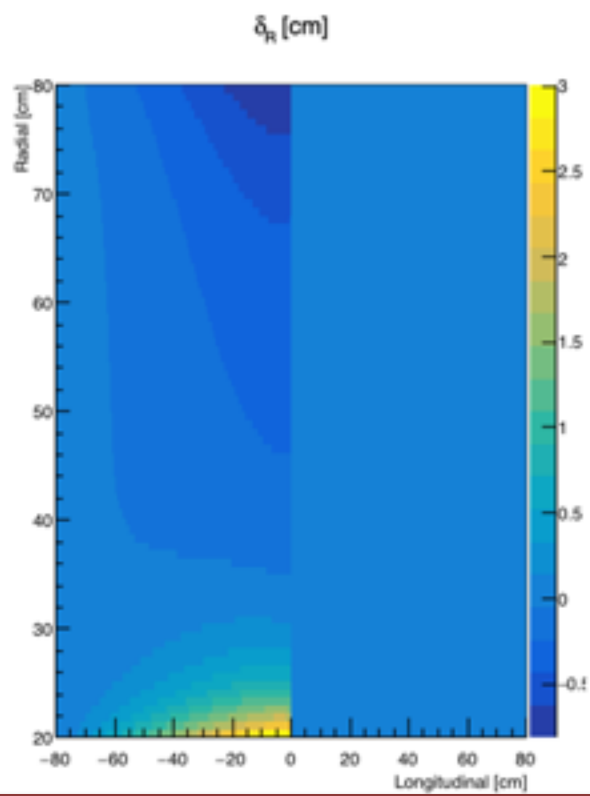
Estimated mean distortions in R

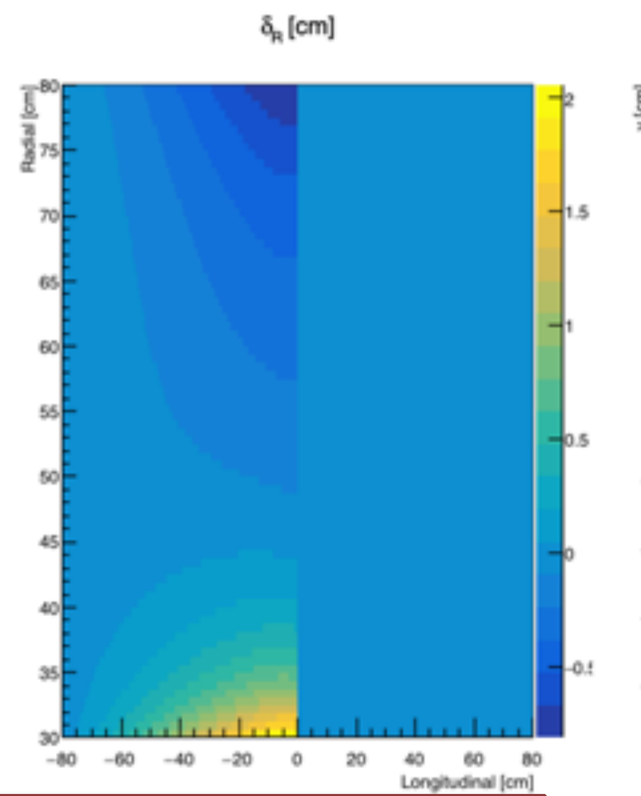
ALICE
Grid size:
Rad = 2.13 cm
Phi = 360 deg
Lon = 2 cm

sPHENIX20
Grid size:
Rad = 0.75 cm
Phi = 360 deg
Lon = 0.64 cm

sPHENIX30
Grid size:
Rad = 0.63 cm
Phi = 360 deg
Lon = 0.64 cm







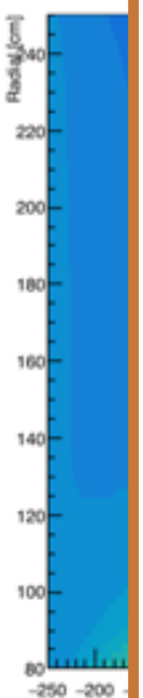
Gas parameters left alike, mean multiplicity scaled by a factor ~2 compared to ALICE

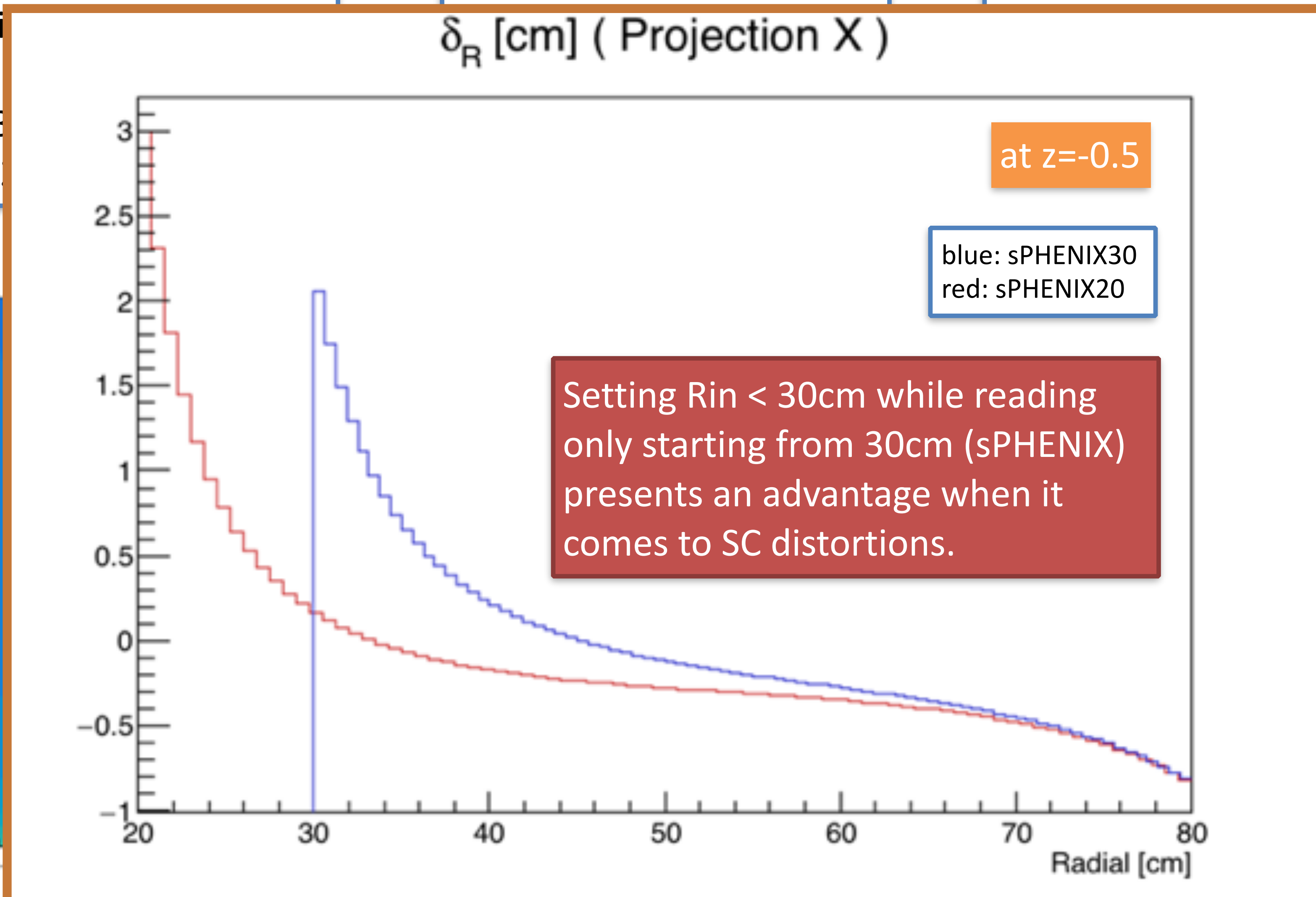
Estimated mean distortions in R

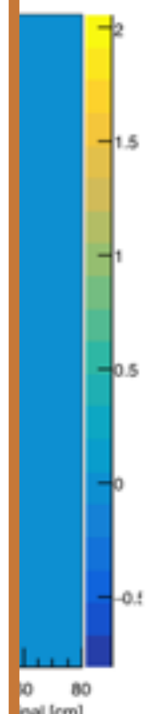
ALICE
Grid si
Rad =
Phi = 3
Lon =

sPHENIX20

sPHENIX30







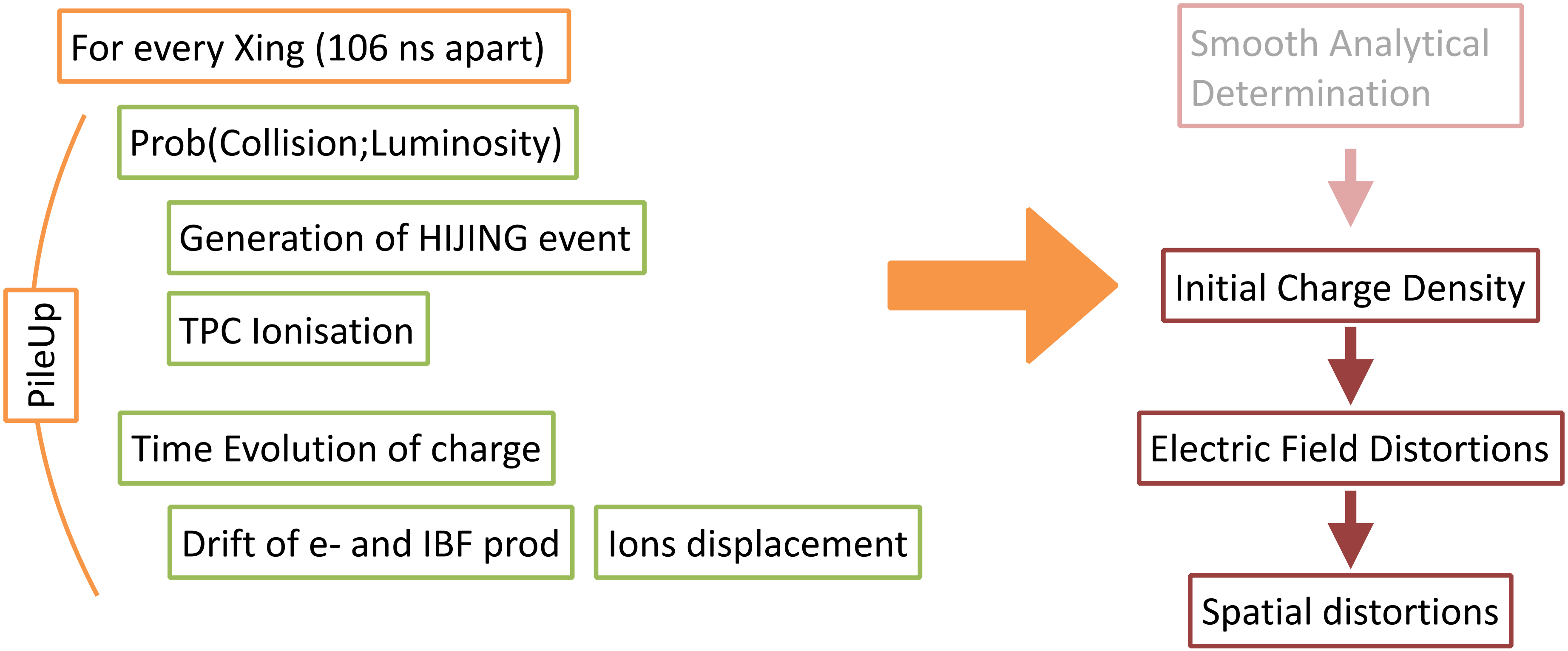
More on Initial Charge Density and the Strategy for Quantification of Residuals

Initial Charge Density: Method 2

- Initial Charge Density was modelled so far using phenomenological expression from ALICE
- As such many control variables like “gas factor”, “multiplicity”, “ion-feedback” are used heuristically.
- To gain full control on the gas response and realistic track density, it is desirable to model this from First Principles.

Very preliminary

Flow Chart for new Initial Charge Density



Collision by collision electron/ion followup to model more accurately the ICD

- Ion latency time period (to account for gas and E field)
- Particle density distribution from MB events from generator

Strategy in Analysis of Distortions

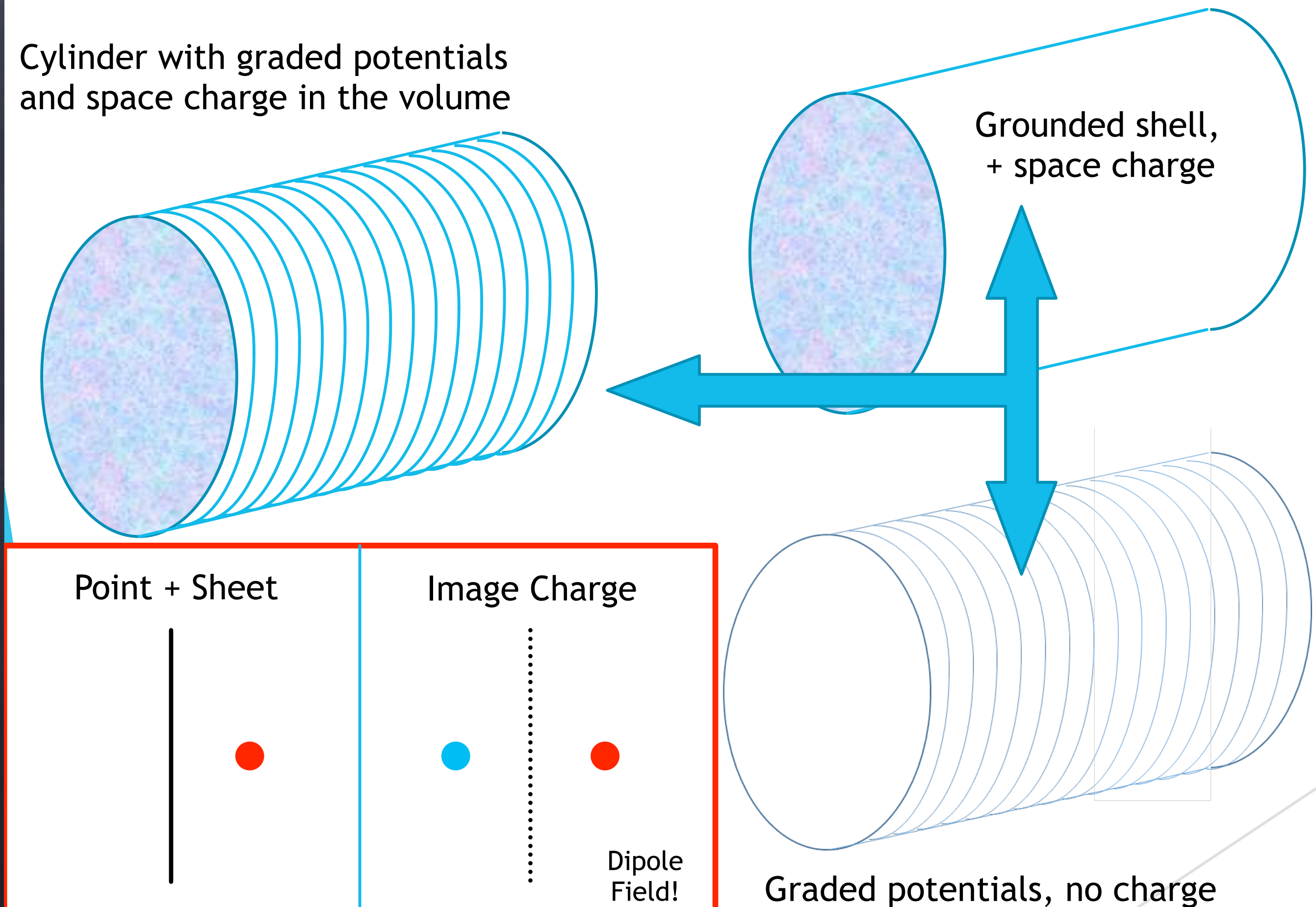
- Determine mean distortions as function of Luminosity
- Determine single event distortions (fluctuations)
  - Particle multiplicity
  - Inaccuracy in Luminosity
  - Inaccuracy in IBF percentage (inaccuracy in gain)

See Alan’s slides (or backup) for the plan of inclusion of these effects into sPHENIX tracking framework.

BACKUP

Tom

Factorization of the Space Charge Problem



- Graded field cage field determined by ANSYS or COLSOL finite element calculations.
- Grounded shell solved using Greene's theorem

$$\Delta G(r, r_{ch}) = \delta(r - r_{ch})$$

$$E_{ch}(r, r_{ch}) = \nabla G(r, r_{ch})$$

$$E = \int \rho(r_{ch}) E_{ch}(r, r_{ch}) dV_{ch}$$

Carlos Tom

Tom

Basic Approach to Solving the Cylinder

- The problem at hand is this: $\Delta G(\vec{x};\vec{x}') = -\delta(\vec{x}-\vec{x}')$, (5.13)

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}\right]G(r,\phi,z;r',\phi',z') = -\frac{\delta(r-r')}{r'}\,\delta(\phi-\phi')\,\delta(z-z'), \quad (5.14)$$

- Our solution begins with solving the homogeneous equation to provide a basis set of functions for the full solution:

$$\Delta\Phi = 0, \qquad \left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+\frac{\partial^2}{\partial z^2}\right)\Phi(r,\phi,z) = 0, \qquad \Phi(r,\phi,z) = R(r)\Phi(\phi)Z(z).$$

Periodicity set m=0,1,2,3,…

$$\Phi_m(\phi) = C_m\, e^{im\phi} = A_m\cos(m\phi) + B_m\sin(m\phi) \qquad \text{with } m \in \mathbb{Z}.$$

$$\frac{R_{rr}}{R}+\frac{1}{r}\frac{R_r}{R}-\frac{m^2}{r^2} = -\frac{Z_{zz}}{Z} = \begin{cases} -\beta^2, & \text{case I;} \\ \beta^2, & \text{case II.} \end{cases}$$

Solution without boundary conditions applied:

$$Z_m(z) = C_m\cosh(\beta z) + D_m\sinh(\beta z),$$
$$R_m(r) = E_m J_m(\beta r) + F_m Y_m(\beta r).$$

Constants formulated to explicitly vanish at r=a

$$R_{mn}(r) = Y_m(\beta_{mn}a)J_m(\beta_{mn}r) - J_m(\beta_{mn}a)Y_m(\beta_{mn}r).$$

Vanishing at r=b forces β to become discreet.

Tom

Finishing the solution

- Once the solutions to the homogeneous equation are known, we express the Dirac delta function in this basis:

$$
\begin{aligned}
\delta(\phi-\phi') &= \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} = \frac{1}{2\pi}\sum_{m=0}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')], \\
\frac{\delta(r-r')}{r} &= \sum_{n=1}^{\infty}\frac{R_{mn}(r)R_{mn}(r')}{\bar{N}_{mn}^2} \quad \text{with} \quad \bar{N}_{nm}^2 = \int_a^b R_{mn}^2(r)\, r dr, \\
m &= 0, 1, 2, \dots .
\end{aligned}
$$

- After which the solution is readily obtained:

$$
G(r,\phi,z;r',\phi',z') = \frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\frac{R_{mn}(r)R_{mn}(r')}{\bar{N}_{mn}^2}\frac{\sinh(\beta_{mn}z_<)\sinh(\beta_{mn}(L-z_>))}{\beta_{mn}\sinh(\beta_{mn}L)}.
$$

- Although the solution is correct, it is not assured to be readily convergent.
- Rossegger used three independent basis sets to obtain stable, differentiable, convergent solutions for the r, ϕ, and z components of the field:

$$
\frac{\partial}{\partial z}G(r,\phi,z,r',\phi',z') = \frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\frac{R_{mn}(r)R_{mn}(r')}{N_{mn}^2}\frac{\partial}{\partial z}\left(\frac{\sinh(\beta_{mn}z_<)\sinh(\beta_{mn}(L-z_>))}{\beta_{mn}\sinh(\beta_{mn}L)}\right), \tag{5.64}
$$

with

$$
\frac{\partial}{\partial z}(\sinh(\beta_{mn}z_<)\sinh(\beta_{mn}(L-z_>))) = \begin{cases} \beta_{mn}\cosh(\beta_{mn}z)\sinh(\beta_{mn}(L-z')), & \text{for } 0 \le z < z' \le L, \\ -\beta_{mn}\cosh(\beta_{mn}(L-z))\sinh(\beta_{mn}z'), & \text{for } 0 \le z' < z \le L. \end{cases}
$$

$$
\frac{\partial}{\partial r}G(r,\phi,z,r',\phi',z') = \frac{1}{\pi L}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}(2-\delta_{m0})\cos[m(\phi-\phi')]\sin(\beta_n z)\sin(\beta_n z')\frac{\partial}{\partial r}\left(\frac{R_{mn\,1}(r_<)R_{mn\,2}(r_>)}{I_m(\beta_n a)K_m(\beta_n b)-I_m(\beta_n b)K_m(\beta_n a)}\right), \tag{5.65}
$$

with

$$
\frac{\partial}{\partial r}(R_{mn\,1}(r_<)R_{mn\,2}(r_>)) = \begin{cases} R'_{mn}(a,r)R_{mn\,2}(r'), & \text{for } a \le r < r' \le b, \\ R_{mn\,1}(r')R'_{mn}(b,r), & \text{for } a \le r' < r \le b, \end{cases}
$$

wherein $R'_{mn}(s,t)$ is

$$
R'_{mn}(s,t) = \frac{\beta_n}{2}\left(K_m(\beta_n s)(I_{m-1}(\beta_n t)+I_{m+1}(\beta_n t)) + I_m(\beta_n s)(K_{m-1}(\beta_n t)+K_{m+1}(\beta_n t)\right).
$$

$$
\frac{\partial}{\partial \phi}G(r,\phi,z,r',\phi',z') = \frac{1}{L}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sin(\beta_n z)\sin(\beta_n z')\frac{R_{nk}(r)R_{nk}(r')}{N_{nk}^2}\frac{\partial}{\partial \phi}\left(\frac{\cosh[\mu_{nk}(\pi-|\phi-\phi'|)]}{\mu_{nk}\sinh(\pi\mu_{nk})}\right) \tag{5.66}
$$

with

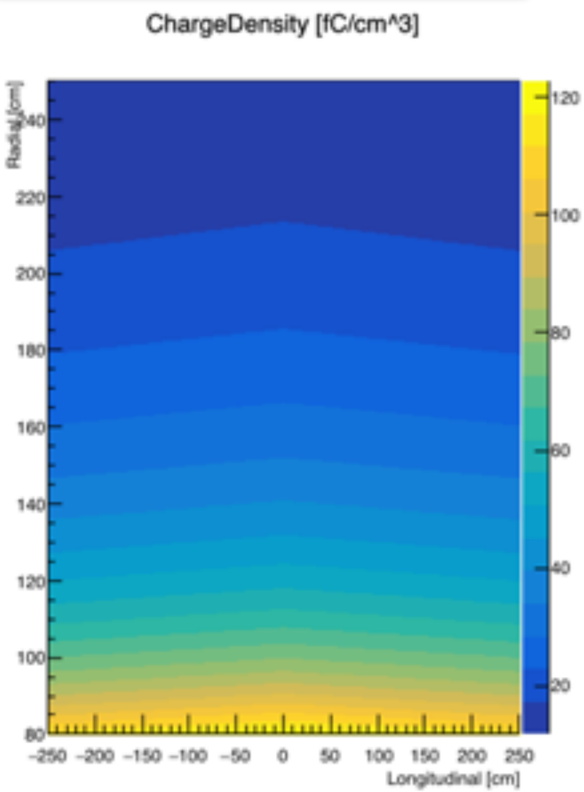
$$
\frac{\partial}{\partial \phi}(\cosh[\mu_{nk}(\pi-|\phi-\phi'|)]) = \begin{cases} -\mu_{nk}\sinh[\mu_{nk}(\pi-(\phi-\phi')), & \text{for } 0 \le \phi' < \phi \le 2\pi \\ \mu_{nk}\sinh[\mu_{nk}(\pi-(\phi'-\phi)), & \text{for } 0 \le \phi < \phi' \le 2\pi \end{cases}
$$

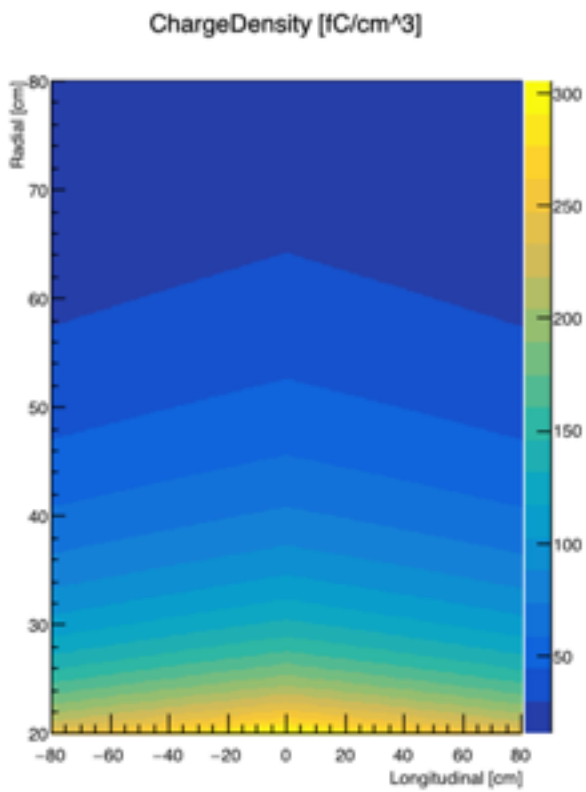
Initial Charge Density

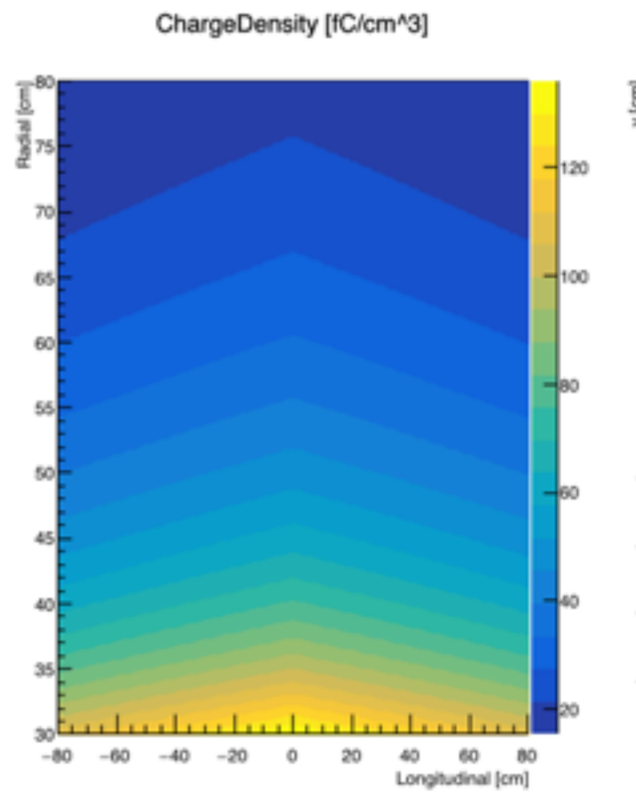
ALICE
Radial dependence set at 2
Gas factor at 1.0/76628.0
Multiplicity at 900
DC Rate at 50kHz
BackFlow at 20 (=1.0%2000)

sPHENIX20
Radial dependence set at 2
Gas factor at 1.0/76628.0
Multiplicity at 450
DC Rate at 50kHz
BackFlow at 6 (=0.3%2000)

sPHENIX30
Radial dependence set at 2
Gas factor at 1.0/76628.0
Multiplicity at 450
DC Rate at 50kHz
BackFlow at 6 (=0.3%2000)







Induced Electric Field

ALICE

Grid size:

Rad = 2.13 cm

Phi = 360 deg

Lon = 2 cm

sPHENIX20

Grid size:

Rad = 0.75 cm

Phi = 360 deg

Lon = 0.64 cm

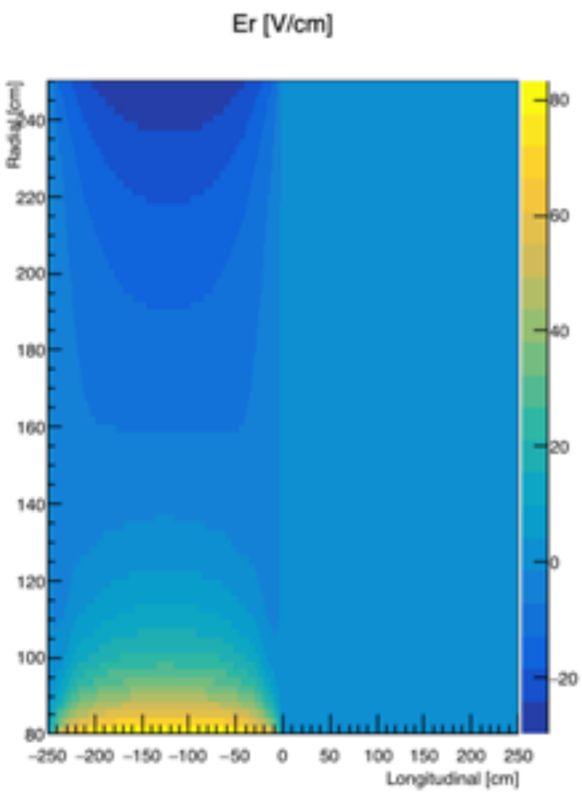
sPHENIX30

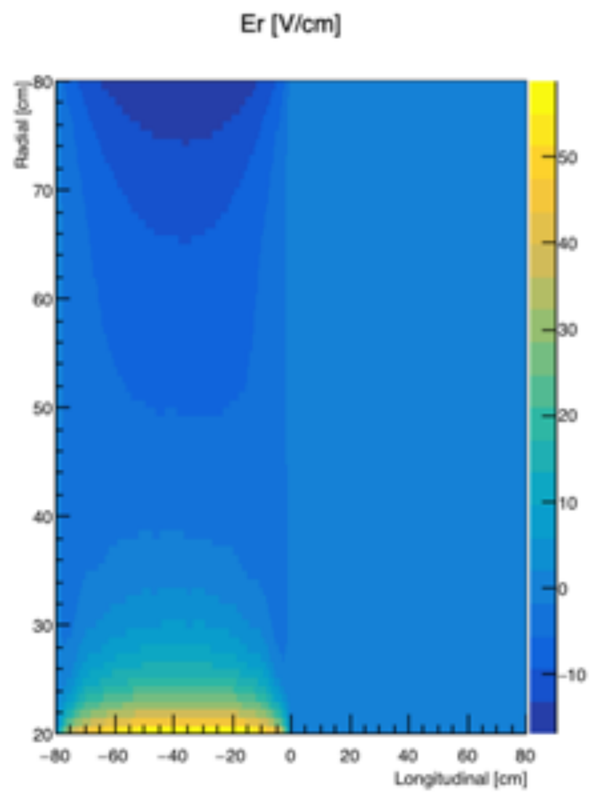
Grid size:

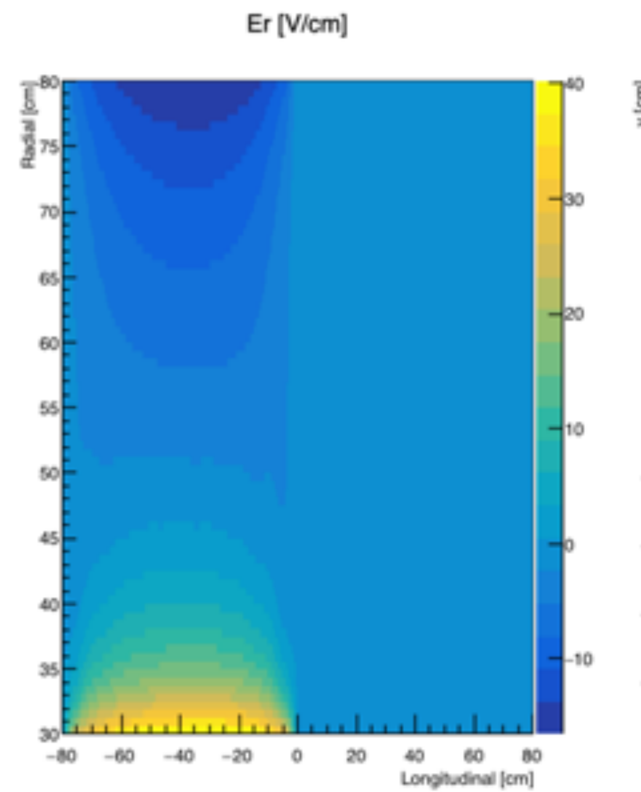
Rad = 0.63 cm

Phi = 360 deg

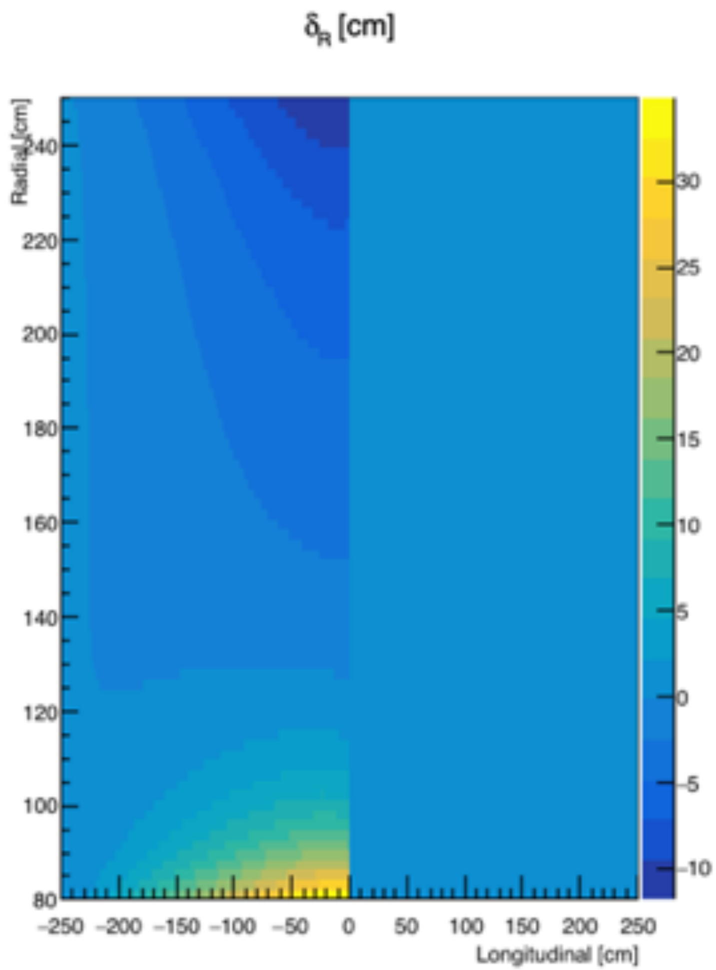
Lon = 0.64 cm

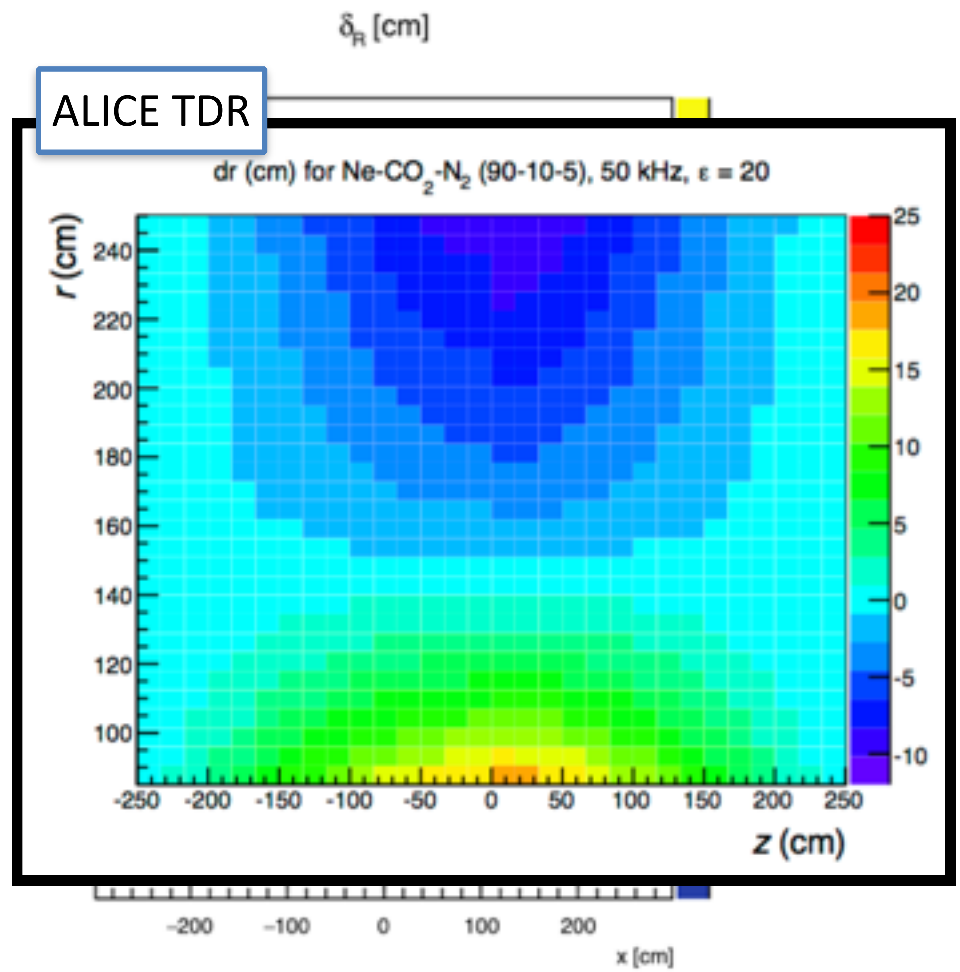




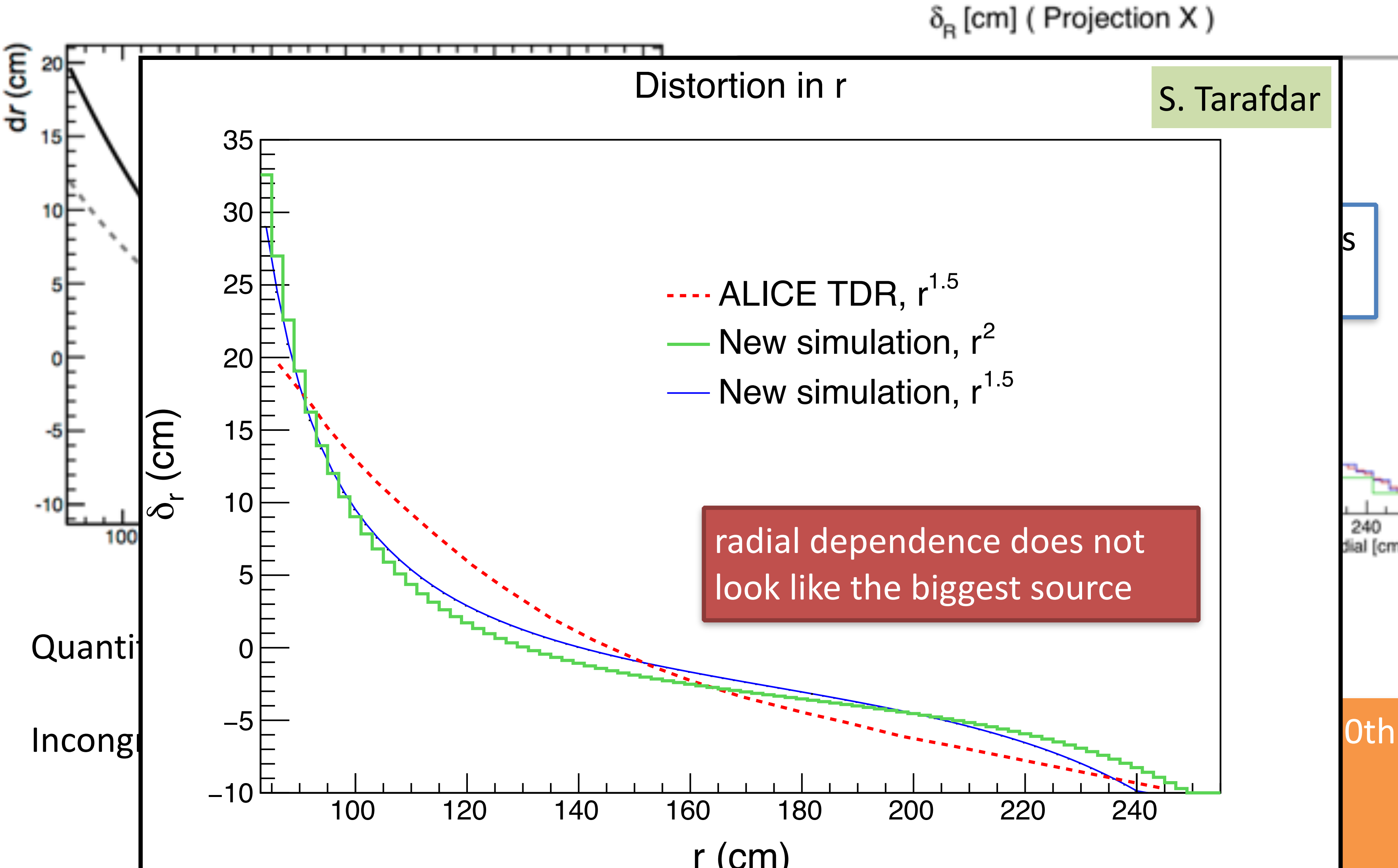


Comparing with ALICE TDR (1/2)





Comparing with ALICE TDR (2/2)



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Traces to pairs

- Ingredients
  - DeltaE for the total track length
  - DeltaE to N ionised electrons

Gas	Ratio	Density*10^{-3} (g/cm^3)	Radiation Length (m)	N_p (cm^{-1})	N_t (cm^{-1})
Ne-CH_4	90-10	0.881	361.8	13.45	44
	80-20	0.862	380.4	14.9	45
	70-30	0.843	401	16.35	46
Ne-C_2H_6	90-10	.0944	344	14.9	49.8
	80-20	0.988	343.9	17.8	56.6
	70-30	1.032	343.4	20.7	63.4
Ne-iC_4H_{10}	90-10	1.06	312	19.2	58.2
	80-20	1.23	285	26.4	73.4
	70-30	1.4	262	33.6	88.6
Ne-CO_2	90-10	1	317	14.35	47.8
	80-20	1.12	293	16.7	52.6
	70-30	1.22	272	19	57.4
Xe-CH_4	90-10	5.34	16.6	42.25	281.6
	80-20	4.83	18.6	40.5	256.2
	70-30	4.31	21.2	38.75	230.8
Xe-C_2H_6	90-10	5.4	16.6	43.7	287.4
	80-20	4.95	18.5	43.4	267.8
	70-30	4.5	21	43.1	248.2
Xe-iC_4H_{10}	90-10	5.53	16.5	48	295.8
	80-20	5.2	18.3	52	284.6
	70-30	4.87	20.6	56	273.4
Xe-CO_2	90-10	5.47	16.5	43.15	285.4
	80-20	5.1	18.4	42.3	263.8
	70-30	4.69	20.7	41.45	242.2

Table 1. (Continued) Parameters of some gas and gas mixtures.

For the moment, I parametrised the number of Nt per cm as cte from this table

Integration into sPHENIX software